Pearson Edexcel

Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE A Level Mathematics
Pure Mathematics Paper 2 (9MA0/02)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is awarded.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.

2. These mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - **bod** – benefit of doubt
   - **ft** – follow through
   - the symbol $\surd$ will be used for correct ft
   - **cao** – correct answer only
   - **cso** - correct solution only. There must be no errors in this part of the question to obtain this mark
   - **isw** – ignore subsequent working
   - **awrt** – answers which round to
   - **SC**: special case
   - **o.e.** – or equivalent (and appropriate)
   - **d** or **dep** – dependent
   - **indep** – independent
   - **dp** decimal places
   - **sf** significant figures
   - ∗ The answer is printed on the paper or ag- answer given

4. All M marks are follow through.

   A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread, however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.

   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but the response is deemed to be valid, examiners must escalate the response for a senior examiner to review.

General Principles for Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\ |pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\ |pq|=|c|\ \text{and}\ |mn|=|a|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to $x=\ldots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

<u>Answers without working</u>

The rubric says that these <u>may</u> not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does <u>not</u> cover this, please contact your team leader for advice.

Question	Scheme	Marks	AOs
1	$\mathrm{g}(x) = \dfrac{2x+5}{x-3},\ x \geq 5$		
(a) **Way 1**	$\mathrm{g}(5) = \dfrac{2(5)+5}{5-3} = 7.5 \Rightarrow \mathrm{gg}(5) = \dfrac{2(\text{"}7.5\text{"})+5}{\text{"}7.5\text{"}-3}$	M1	1.1b
	$\mathrm{gg}(5) = \dfrac{40}{9}\ \left(\text{or } 4\dfrac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(a) **Way 2**	$\mathrm{gg}(x) = \dfrac{2\left(\dfrac{2x+5}{x-3}\right)+5}{\left(\dfrac{2x+5}{x-3}\right)-3} \Rightarrow \mathrm{gg}(5) = \dfrac{2\left(\dfrac{2(5)+5}{(5)-3}\right)+5}{\left(\dfrac{2(5)+5}{(5)-3}\right)-3}$	M1	1.1b
	$\mathrm{gg}(5) = \dfrac{40}{9}\ \left(\text{or } 4\dfrac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(b)	$\{\text{Range:}\}\ \ 2 < y \leq \dfrac{15}{2}$	B1	1.1b
		(1)	
(c) **Way 1**	$y = \dfrac{2x+5}{x-3} \Rightarrow yx - 3y = 2x + 5 \Rightarrow yx - 2x = 3y + 5$	M1	1.1b
	$x(y-2) = 3y+5 \Rightarrow x = \dfrac{3y+5}{y-2}\ \left\{\text{or } y = \dfrac{3x+5}{x-2}\right\}$	M1	2.1
	$\mathrm{g}^{-1}(x) = \dfrac{3x+5}{x-2},\quad 2 < x \leq \dfrac{15}{2}$	A1ft	2.5
		(3)	
(c) **Way 2**	$y = \dfrac{2x-6+11}{x-3} \Rightarrow y = 2 + \dfrac{11}{x-3} \Rightarrow y - 2 = \dfrac{11}{x-3}$	M1	1.1b
	$x - 3 = \dfrac{11}{y-2} \Rightarrow x = \dfrac{11}{y-2} + 3\ \left\{\text{or } y = \dfrac{11}{x-2} + 3\right\}$	M1	2.1
	$\mathrm{g}^{-1}(x) = \dfrac{11}{x-2} + 3,\quad 2 < x \leq \dfrac{15}{2}$	A1ft	2.5
		(3)	
			(6 marks)

Notes for Question 1

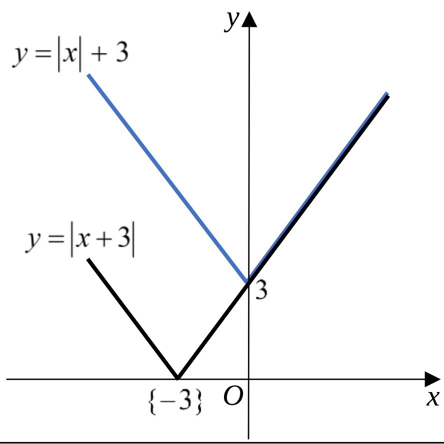
(a)	
M1:	Full method of attempting $\mathrm{g}(5)$ and substituting the result into g
Note:	**Way 2:** Attempts to substitute $x = 5$ into $\dfrac{2\left(\dfrac{2x+5}{x-3}\right)+5}{\left(\dfrac{2x+5}{x-3}\right)-3}$, o.e. Note that $\mathrm{gg}(x) = \dfrac{9x-5}{14-x}$
A1:	Obtains $\dfrac{40}{9}$ or $4\dfrac{4}{9}$ or $4.\dot{4}$ or an exact equivalent
Note:	Give A0 for 4.4 or 4.444... without reference to $\dfrac{40}{9}$ or $4\dfrac{4}{9}$ or $4.\dot{4}$

Notes for Question 1 Continued	
(b)	
B1:	States $2 < y \leq \frac{15}{2}$ Accept any of $2 < \text{g} \leq \frac{15}{2}$, $2 < \text{g}(x) \leq \frac{15}{2}$, $\left(2, \frac{15}{2}\right]$
Note:	Accept $\text{g}(x) > 2$ **and** $\text{g}(x) \leq \frac{15}{2}$ o.e.
(c) Way 1	
M1:	Correct method of cross multiplication followed by an attempt to collect terms in *x* or terms in a swapped *y*
M1:	A complete method (i.e. as above and also factorising and dividing) to find the inverse
A1ft:	Uses correct notation to correctly define the inverse function g^{-1}, where the domain of g^{-1} stated correctly or correctly followed through (using correct notation) on the values shown in their range in part (b). Allow $\text{g}^{-1} : x \to$. Condone $\text{g}^{-1} = ...$ Do not accept $y = ...$
Note:	Correct notation is required when stating the domain of $\text{g}^{-1}(x)$. Allow $2 < x \leq \frac{15}{2}$ or $\left(2, \frac{15}{2}\right]$ Do not allow any of e.g. $2 < \text{g} \leq \frac{15}{2}$, $2 < \text{g}^{-1}(x) \leq \frac{15}{2}$
Note:	Do not allow A1ft for following through their range in (b) to give a domain for g^{-1} as $x \in \mathbb{R}$
(c) Way 2	
M1:	Writes $y = \frac{2x+5}{x-3}$ in the form $y = 2 \pm \frac{k}{x-3}$, $k \neq 0$ and rearranges to isolate *y* and 2 on one side of their equation. **Note:** Allow the equivalent method with *x* swapped with *y*
M1:	A complete method to find the inverse
A1ft:	As in Way 1
Note:	If a candidate scores no marks in part (c), but • states the domain of g^{-1} correctly, **or** • states a domain of g^{-1} which is correctly followed through on the values shown in their range in part (b) then give special case (SC) M1 M0 A0

Question	Scheme	Marks	AOs
2	$\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OC} = a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$, $a < 0$ $\overrightarrow{AB} = \overrightarrow{BD}$, $\lvert\overrightarrow{AB}\rvert = 4$		
(a)	E.g. $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AB}$ **or** $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AB} = \overrightarrow{OB} + \overrightarrow{OB} - \overrightarrow{OA} = 2\overrightarrow{OB} - \overrightarrow{OA}$ **or** $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AB}$		
	$= \begin{pmatrix}4\\-2\\3\end{pmatrix} + \begin{pmatrix}4\\-2\\3\end{pmatrix} - \begin{pmatrix}2\\3\\-4\end{pmatrix} \left\{ = \begin{pmatrix}4\\-2\\3\end{pmatrix} + \begin{pmatrix}2\\-5\\7\end{pmatrix} \right\}$ **or** $= \begin{pmatrix}2\\3\\-4\end{pmatrix} + 2\left(\begin{pmatrix}4\\-2\\3\end{pmatrix} - \begin{pmatrix}2\\3\\-4\end{pmatrix}\right) \left\{ = \begin{pmatrix}2\\3\\-4\end{pmatrix} + 2\begin{pmatrix}2\\-5\\7\end{pmatrix} \right\}$	M1	3.1a
	$= \begin{pmatrix}6\\-7\\10\end{pmatrix}$ or $6\mathbf{i} - 7\mathbf{j} + 10\mathbf{k}$	A1	1.1b
		(2)	
(b)	$(a-2)^2 + (5-3)^2 + (-2--4)^2$	M1	1.1b
	$\left\{\lvert\overrightarrow{AC}\rvert = 4 \Rightarrow\right\} (a-2)^2 + (5-3)^2 + (-2--4)^2 = (4)^2$ $\Rightarrow (a-2)^2 = 8 \Rightarrow a = ...$ **or** $\Rightarrow a^2 - 4a - 4 = 0 \Rightarrow a = ...$	dM1	2.1
	(as $a < 0 \Rightarrow$) $a = 2 - 2\sqrt{2}$ (or $a = 2 - \sqrt{8}$)	A1	1.1b
		(3)	
			(5 marks)

Notes for Question 2

(a)	
M1:	Complete ***applied*** strategy to find a vector expression for $\overrightarrow{OD}$
A1:	See scheme
Note:	Give M0 for subtracting the wrong way wrong to give e.g. $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + (2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) - (4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) = (4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + (-2\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}) = (2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$
Note:	Writing e.g. $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{AB}$ or $\overrightarrow{OD} = 2\overrightarrow{OB} - \overrightarrow{OA}$ with no other work is M0
Note:	Finding ***coordinates***, i.e. $(6, -7, 10)$ without reference to the correct position vectors is A0
Note:	Allow M1A1 for writing down $6\mathbf{i} - 7\mathbf{j} + 10\mathbf{k}$ with no working
Note:	M1 can be implied for at least two correct components in their position vector of D
(b)	
M1:	Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OC}$, then squares and adds each of the 3 components **Note:** Ignore labelling
dM1:	Complete method of ***correctly*** applying Pythagoras' Theorem on $\lvert\overrightarrow{AC}\rvert = 4$ and using a correct method of solving their resulting quadratic equation to find at least one of $a = ...$
Note:	Condone at least one of either awrt 4.8 or awrt -0.83 for the dM mark
A1:	Obtains **only one** exact value, $a = 2 - 2\sqrt{2}$
Note:	Writing $a = 2 \pm 2\sqrt{2}$, without evidence of rejecting $a = 2 + 2\sqrt{2}$ is A0
Note:	Allow exact alternatives such as $2 - \sqrt{8}$ or $\dfrac{4 - \sqrt{32}}{2}$ for A1, and isw can be applied
Note:	Writing $a = -0.828...$, without reference to a correct exact value is A0

Question	Scheme		Marks	AOs
3	Statement: "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."			
(a)	E.g. $m = \sqrt{3},\ n = \sqrt{12}$		M1	1.1b
	$\{mn =\}\ (\sqrt{3})(\sqrt{12}) = 6$ $\Rightarrow$ statement untrue **or** 6 is not irrational or 6 is rational		A1	2.4
			(2)	
(b)(i), (ii) Way 1		V shaped graph {reasonably} symmetrical about the y-axis with vertical interpret $(0, 3)$ or 3 stated or marked on the positive y-axis	B1	1.1b
		Superimposes the graph of $y = \lvert x+3\rvert$ on top of the graph of $y = \lvert x\rvert + 3$	M1	3.1a
	the graph of $y = \lvert x\rvert + 3$ is either the same or above the graph of $y = \lvert x+3\rvert$ {for corresponding values of x} **or** when $x \geq 0$, both graphs are equal (or the same) when $x < 0$, the graph of $y = \lvert x\rvert + 3$ is above the graph of $y = \lvert x+3\rvert$		A1	2.4
			(3)	
(b)(ii) Way 2	**<u>Reason 1</u>** When $x \geq 0$, $\lvert x\rvert + 3 = \lvert x+3\rvert$ **<u>Reason 2</u>** When $x < 0$, $\lvert x\rvert + 3 > \lvert x+3\rvert$	Any one of Reason 1 or Reason 2	M1	3.1a
		Both Reason 1 and Reason 2	A1	2.4
			(5 marks)	

Notes for Question 3

(a)	
M1:	States or uses any pair of ***different*** numbers that will disprove the statement. E.g. $\sqrt{3},\ \sqrt{12}$; $\sqrt{2},\ \sqrt{8}$; $\sqrt{5}, -\sqrt{5}$; $\frac{1}{\pi}, 2\pi$; $3\text{e}, \frac{4}{5\text{e}}$;
A1:	Uses correct reasoning to disprove the given statement, with a correct conclusion
Note:	Writing $(3\text{e})\left(\frac{4}{5\text{e}}\right) = \frac{12}{5} \Rightarrow$ untrue is sufficient for M1A1
(b)(i)	
B1:	See scheme
(b)(ii)	
M1:	For constructing a method of comparing $\lvert x\rvert + 3$ with $\lvert x+3\rvert$. See scheme.
A1:	Explains fully why $\lvert x\rvert + 3 \geq \lvert x+3\rvert$. See scheme.
Note:	Do not allow either $x > 0,\ \lvert x\rvert + 3 \geq \lvert x+3\rvert$ or $x \geq 0,\ \lvert x\rvert + 3 \geq \lvert x+3\rvert$ as a valid reason
Note	$x = 0$ (or where necessary $x = -3$) need to be considered in their solutions for A1
Note:	Do not allow an incorrect statement such as $x \leq 0,\ \lvert x\rvert + 3 > \lvert x+3\rvert$ for A1

Notes for Question 3 Continued			
(b)(ii)			
Note:	Allow M1A1 for $x > 0,\ \lvert x\rvert + 3 = \lvert x+3\rvert$ and for $x \le 0,\ \lvert x\rvert + 3 \ge \lvert x+3\rvert \geqslant$		
Note:	Allow M1 for any of • x is positive, $\lvert x\rvert + 3 = \lvert x+3\rvert$ • x is negative, $\lvert x\rvert + 3 > \lvert x+3\rvert$ • $x > 0,\ \lvert x\rvert + 3 = \lvert x+3\rvert$ • $x \le 0,\ \lvert x\rvert + 3 \ge \lvert x+3\rvert$ • $x > 0,\ \lvert x\rvert + 3$ and $\lvert x+3\rvert$ are equal • $x \ge 0,\ \lvert x\rvert + 3$ and $\lvert x+3\rvert$ are equal • when $x \ge 0$, both graphs are equal • for positive values $\lvert x\rvert + 3$ and $\lvert x+3\rvert$ are the same Condone for M1 • $x \le 0,\ \lvert x\rvert + 3 > \lvert x+3\rvert$ • $x < 0,\ \lvert x\rvert + 3 \ge \lvert x+3\rvert$		
(b)(ii) Way 3	• For $x > 0,\ \lvert x\rvert + 3 = \lvert x+3\rvert$ • For $-3 < x < 0$, as $\lvert x\rvert + 3 > 3$ and $\{0 <\}\ \lvert x+3\rvert < 3$, then $\lvert x\rvert + 3 > \lvert x+3\rvert$	M1	3.1a
	• For $x \le -3$, as $\lvert x\rvert + 3 = -x+3$ and $\lvert x+3\rvert = -x-3$, then $\lvert x\rvert + 3 > \lvert x+3\rvert$	A1	2.4

Question	Scheme	Marks	AOs
4	(i) $\sum_{r=1}^{16}\left(3+5r+2^r\right)=131\,798$; (ii) $u_1, u_2, u_3, ...,: u_{n+1}=\frac{1}{u_n}, u_1=\frac{2}{3}$		
(i) Way 1	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\}\sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=\frac{16}{2}(2(8)+15(5))\ +\ \frac{2(2^{16}-1)}{2-1}$	M1 M1	1.1b 1.1b
	$=728+131\,070\ =\ 131\,798$ *	A1*	2.1
		(4)	
(i) Way 2	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\}\sum_{r=1}^{16}3\ +\ \sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=(3\times16)+\frac{16}{2}(2(5)+15(5))\ +\ \frac{2(2^{16}-1)}{2-1}$	M1 M1	1.1b 1.1b
	$=48+680+131\,070\ =\ 131\,798$ *	A1*	2.1
		(4)	
(i) Way 3	Sum $=\ 10+17+26+39+60+97+166+299+560+1077+2106$ $+4159+8260+16457+32846+65619=131\,798$ *	M1 M1 M1 A1*	3.1a 1.1b 1.1b 2.1
		(4)	
(ii)	$\left\{u_1=\frac{2}{3}\right\},\ u_2=\frac{3}{2},\ u_3=\frac{2}{3}, ...$ **(*can be implied by later working*)**	M1	1.1b
	$\left\{\sum_{r=1}^{100}u_r=\right\}\ 50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$	M1	2.2a
	$=\frac{325}{3}\ \left(\text{or } 108\frac{1}{3} \text{ or } 108.\dot{3} \text{ or } \frac{1300}{12} \text{ or } \frac{650}{6}\right)$	A1	1.1b
		(3)	
			(7 marks)

Notes for Question 4	
(i)	
M1:	Uses a correct methodical strategy to enable the given sum, $\sum_{r=1}^{16}(3+5r+2^r)$ to be found Allow M1 for any of the following: • expressing the given sum as either $\sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}(2^r)$, $\sum_{r=1}^{16}3+\sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}(2^r)$ or $\sum_{r=1}^{16}3 + 5\sum_{r=1}^{16}r + \sum_{r=1}^{16}(2^r)$ • attempting to find both $\sum_{r=1}^{16}(3+5r)$ and $\sum_{r=1}^{16}(2^r)$ separately • (3×16) and attempting to find both $\sum_{r=1}^{16}(5r)$ and $\sum_{r=1}^{16}(2^r)$ separately
M1:	**Way 1:** Correct method for finding the sum of an AP with $a=8, d=5, n=16$ **Way 2:** (3×16) and a correct method for finding the sum of an AP
M1:	Correct method for finding the sum of a GP with $a=2, r=2, n=16$
A1*:	For all steps fully shown (with correct formulae used) leading to $131\,798$
Note:	**Way 1:** Give 2nd M1 for writing $\sum_{r=1}^{16}(3+5r)$ as $\frac{16}{2}(8+83)$
Note:	**Way 2:** Give 2nd M1 for writing $\sum_{r=1}^{16}3 + \sum_{r=1}^{16}(5r)$ as $48+\frac{16}{2}(5+80)$ or $48+680$
Note:	Give 3rd M1 for writing $\sum_{r=1}^{16}(2^r)$ as $\frac{2(1-2^{16})}{1-2}$ or $2(2^{16}-1)$ or $(2^{17}-2)$
(i)	
Way 3	
M1:	At least 6 correct terms **and** 16 terms shown
M1:	At least 10 correct terms (may not be 16 terms)
M1:	At least 15 correct terms (may not be 16 terms)
A1*:	All 16 terms correct **and** an indication that the sum is $131\,798$
(ii)	
M1:	For some indication that the next two terms of this sequence are $\frac{3}{2}, \frac{2}{3}$
M1:	For deducing that the sum can be found by applying $50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$, o.e.
A1:	Obtains $\frac{325}{3}$ or $108\frac{1}{3}$ or $108.\dot{3}$ or an exact equivalent
Note:	Allow 1st M1 for $u_2=\frac{3}{2}$ (or equivalent) **and** $u_3=\frac{2}{3}$ (or equivalent)
Note:	Allow 1st M1 for the first 3 terms written as $\frac{2}{3}, \frac{3}{2}, \frac{2}{3}, \ldots$
Note:	Allow 1st M1 for the 2nd and 3rd terms written as $\frac{3}{2}, \frac{2}{3}, \ldots$ in the correct order
Note:	Condone $\frac{2}{3}$ written as 0.66 or awrt 0.67 for the 1st M1 mark
Note:	Give A0 for 108.3 or 108.333... without reference to $\frac{325}{3}$ or $108\frac{1}{3}$ or $108.\dot{3}$

Question	Scheme	Marks	AOs
5	The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root		
(a)	$\left\{f(x) = 2x^3 + x^2 - 1 \Rightarrow\right\}\ f'(x) = 6x^2 + 2x$	B1	1.1b
	$\left\{x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} \Rightarrow\right\}\ \{x_{n+1}\} = x_n - \dfrac{2x_n^{\ 3} + x_n^{\ 2} - 1}{6x_n^{\ 2} + 2x_n}$	M1	1.1b
	$= \dfrac{x_n(6x_n^{\ 2} + 2x_n) - (2x_n^{\ 3} + x_n^{\ 2} - 1)}{6x_n^{\ 2} + 2x_n} \Rightarrow x_{n+1} = \dfrac{4x_n^{\ 3} + x_n^{\ 2} + 1}{6x_n^{\ 2} + 2x_n}$ *	A1*	2.1
		(3)	
(b)	$\{x_1 = 1 \Rightarrow\}\ x_2 = \dfrac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)}$ or $x_2 = 1 - \dfrac{2(1)^3 + (1)^2 - 1}{6(1)^2 + 2(1)}$	M1	1.1b
	$\Rightarrow x_2 = \dfrac{3}{4},\ x_3 = \dfrac{2}{3}$	A1	1.1b
		(2)	
(c)	Accept any reasons why the Newton-Raphson **method** cannot be used with $x_1 = 0$ which refer or ***allude*** to either the stationary point or the tangent. E.g. • There is a stationary point at $x = 0$ • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) would not meet the *x*-axis • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) is horizontal	B1	2.3
		(1)	
			(6 marks)

Notes for Question 5

(a)	
B1:	States that $f'(x) = 6x^2 + 2x$ or states that $f'(x_n) = 6x_n^{\ 2} + 2x_n$ (Condone $\dfrac{dy}{dx} = 6x^2 + 2x$)
M1:	Substitutes $f(x_n) = 2x_n^{\ 3} + x_n^{\ 2} - 1$ and their $f'(x_n)$ into $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
A1*:	A correct intermediate step of making a common denominator which leads to the given answer
Note:	Allow B1 if $f'(x) = 6x^2 + 2x$ is applied as $f'(x_n)$ (or $f'(x)$) in the NR formula $\{x_{n+1}\} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
Note:	Allow M1A1 for • $x_{n+1} = x - \dfrac{2x^3 + x^2 - 1}{6x^2 + 2x} = \dfrac{x(6x^2 + 2x) - (2x^3 + x^2 - 1)}{6x^2 + 2x} \Rightarrow x_{n+1} = \dfrac{4x_n^{\ 3} + x_n^{\ 2} + 1}{6x_n^{\ 2} + 2x_n}$
Note	Condone $x = x - \dfrac{2x^3 + x^2 - 1}{"6x^2 + 2x"}$ for M1
Note	Condone $x_n - \dfrac{2x_n^{\ 3} + x_n^{\ 2} - 1}{"6x_n^{\ 2} + 2x_n"}$ or $x - \dfrac{2x^3 + x^2 - 1}{"6x^2 + 2x"}$ (i.e. no $x_{n+1} = \ldots$) for M1
Note:	Give M0 for $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ followed by $x_{n+1} = 2x_n^{\ 3} + x_n^{\ 2} - 1 - \dfrac{2x_n^{\ 3} + x_n^{\ 2} - 1}{6x_n^{\ 2} + 2x_n}$
Note:	Correct notation, i.e. x_{n+1} and x_n must be seen in their final answer for A1*

Notes for Question 5 Continued	
(b)	
M1:	An attempt to use the given or their formula once. Can be implied by $\frac{4(1)^3+(1)^2+1}{6(1)^2+2(1)}$ or 0.75 o.e.
Note:	Allow one slip in substituting $x_1=1$
A1:	$x_2=\frac{3}{4}$ and $x_3=\frac{2}{3}$
Note:	Condone $x_2=\frac{3}{4}$ and $x_3=$ awrt 0.667 for A1
Note:	Condone $\frac{3}{4}, \frac{2}{3}$ listed in a correct order ignoring subscripts
(c)	
B1:	See scheme
Note:	Give B0 for the following isolated reasons: e.g. • You cannot divide by 0 • The fraction (or the NR formula) is undefined at $x=0$ • At $x=0$, $f'(x_1)=0$ • x_1 cannot be 0 • $6x^2+2x$ cannot be 0 • the denominator is 0 which cannot happen • if $x_1=0$, $6x^2+2x=0$

Question	Scheme	Marks	AOs
6	(a) $\mathrm{f}(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$		
(a)	(i) $\{\mathrm{f}(2) = -24 + 32 - 18 + 10 \Rightarrow\}\ \mathrm{f}(2) = 0$	B1	1.1b
	(ii) $\{\mathrm{f}(x) =\}\ (x-2)(-3x^2 + 2x - 5)$ or $(2-x)(3x^2 - 2x + 5)$	M1	2.2a
		A1	1.1b
		(3)	
(b)	$-3y^6 + 8y^4 - 9y^2 + 10 = 0 \Rightarrow (y^2 - 2)(-3y^4 + 2y^2 - 5) = 0$		
	Gives a partial explanation by • explaining that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions with a reason, e.g. $b^2 - 4ac = (2)^2 - 4(-3)(-5) = -56 < 0$ • or stating that $y^2 = 2$ has 2 {real} solutions or $y = \pm\sqrt{2}$ {only}	M1	2.4
	Complete proof that the given equation has exactly two {real} solutions	A1	2.1
		(2)	
(c)	$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0;\ 7\pi \le \theta < 10\pi$		
	{Deduces that} there are 3 solutions	B1	2.2a
		(1)	
			(6 marks)

Notes for Question 6

(a)(i)	
B1:	$\mathrm{f}(2) = 0$ or 0 stated by itself in part (a)(i)
(a)(ii)	
M1:	Deduces that $(x-2)$ or $(2-x)$ is a factor and attempts to find the other quadratic factor by • using long division to obtain either $\pm 3x^2 \pm kx + ...,\ k = \text{value} \ne 0$ or $\pm 3x^2 \pm \alpha x + \beta,\ \beta = \text{value} \ne 0,\ \alpha$ can be 0 • factorising to obtain their quadratic factor in the form $(\pm 3x^2 \pm kx \pm c),\ k = \text{value} \ne 0$, c can be 0, or in the form $(\pm 3x^2 \pm \alpha x \pm \beta),\ \beta = \text{value} \ne 0,\ \alpha$ can be 0
A1:	$(x-2)(-3x^2 + 2x - 5),\ (2-x)(3x^2 - 2x + 5)$ or $-(x-2)(3x^2 - 2x + 5)$ stated together as a product
(b)	
M1:	See scheme
A1:	See scheme. Proof must be correct ***with no errors***, e.g. giving an incorrect discriminant value
Note:	Correct calculation e.g. $(2)^2 - 4(-3)(-5)$, $4 - 60$ or -56 must be given for the first explanation
Note:	Note that M1 can be allowed for • a correct follow through calculation for the discriminant of their "$-3y^4 + 2y^2 - 5$" which would lead to a value < 0 together with an explanation that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions • **or** for the omission of < 0
Note:	< 0 must also been stated in a discriminant method for A1
Note:	Do not allow A1 for incorrect working, e.g. $(2)^2 - 4(-3)(-5) = -54 < 0$
Note:	$y^2 = 2 \Rightarrow y = \pm 2$, so 2 solutions is not allowed for A1, but can be condoned for M1
Note:	Using the formula on $-3y^4 + 2y^2 - 5 = 0$ or $-3x^2 + 2x - 5 = 0$ gives y^2 or $x = \dfrac{-2 \pm \sqrt{-56}}{-6}$ or $\dfrac{-1 \pm \sqrt{-14}}{-3}$

Notes for Question 6 Continued	
Note:	Completing the square on $-3x^2+2x-5=0$ gives $x^2-\frac{2}{3}x+\frac{5}{3}=0 \Rightarrow \left(x-\frac{1}{3}\right)^2-\frac{1}{9}+\frac{5}{3}=0 \Rightarrow x=\frac{1}{3}\pm\sqrt{\frac{-14}{9}}$
Note:	Do not recover work for part (b) in part (c)
(c)	
B1:	See scheme
Note:	Give B0 for stating $\theta=$ awrt 23.1, awrt 26.2, awrt 29.4 **without** reference to 3 solutions

Question	Scheme	Marks	AOs
7	(i) $4\sin x = \sec x,\ 0 \le x < \dfrac{\pi}{2}$; (ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$		
(i) Way 1	For $\sec x = \dfrac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 2\sin 2x = 1 \Rightarrow \sin 2x = \dfrac{1}{2}$	M1	3.1a
	$x = \dfrac{1}{2}\arcsin\left(\dfrac{1}{2}\right)$ or $\dfrac{1}{2}\left(\pi - \arcsin\left(\dfrac{1}{2}\right)\right) \Rightarrow x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(i) Way 2	For $\sec x = \dfrac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 16\sin^2 x\cos^2 x = 1$ $16\sin^2 x(1-\sin^2 x) = 1$ | $16(1-\cos^2 x)\cos^2 x = 1$ $16\sin^4 x - 16\sin^2 x + 1 = 0$ | $16\cos^4 x - 16\cos^2 x + 1 = 0$ $\sin^2 x$ or $\cos^2 x = \dfrac{16 \pm \sqrt{192}}{32}$ $\left\{= \dfrac{2 \pm \sqrt{3}}{4}\right.$ or $\left. 0.933..., 0.066...\right\}$	M1	3.1a
	$x = \arcsin\left(\sqrt{\dfrac{2 \pm \sqrt{3}}{4}}\right)$ or $x = \arccos\left(\sqrt{\dfrac{2 \pm \sqrt{3}}{4}}\right) \Rightarrow x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(ii)	Complete strategy, i.e. • Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\sin(\theta - \alpha) = 2$, finds both R and α, and proceeds to $\sin(\theta - \alpha) = k,\ \lvert k\rvert < 1,\ k \ne 0$ • Applies $(5\sin\theta - 5\cos\theta)^2 = 2^2$, followed by applying both $\cos^2\theta + \sin^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to proceed to $\sin 2\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$	M1	3.1a
	$R = \sqrt{50}$ $\tan\alpha = 1 \Rightarrow \alpha = 45°$ | $(5\sin\theta - 5\cos\theta)^2 = 2^2 \Rightarrow$ $25\sin^2\theta + 25\cos^2\theta - 50\sin\theta\cos\theta = 4$ $\Rightarrow 25 - 25\sin 2\theta = 4$	M1	1.1b
	$\sin(\theta - 45°) = \dfrac{2}{\sqrt{50}}$ | $\sin 2\theta = \dfrac{21}{25}$	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arcsin\left(\dfrac{2}{\sqrt{50}}\right) + 45°$ | e.g. $\theta = \dfrac{1}{2}\left(\arcsin\left(\dfrac{21}{25}\right)\right)$	dM1	1.1b
	$\theta = \text{awrt } 61.4°,\ \text{awrt } 208.6°$	A1	2.1
	Note: Working in radians does not affect any of the first 4 marks		
		(5)	
			(9 marks)

Question	Scheme		Marks	AOs
7	(ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$			
(ii) Alt 1	Complete strategy, i.e. • Attempts to apply $(5\sin\theta)^2 = (2+5\cos\theta)^2$ or $(5\sin\theta - 2)^2 = (5\cos\theta)^2$ followed by applying $\cos^2\theta + \sin^2\theta = 1$ and solving a quadratic equation in either $\sin\theta$ or $\cos\theta$ to give at least one of $\sin\theta = k$ or $\cos\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$		M1	3.1a
	e.g. $25\sin^2\theta = 4 + 20\cos\theta + 25\cos^2\theta$ $\Rightarrow 25(1-\cos^2\theta) = 4 + 20\cos\theta + 25\cos^2\theta$ **or** e.g. $25\sin^2\theta - 20\sin\theta + 4 = 25\cos^2\theta$ $\Rightarrow 25\sin^2\theta - 20\sin\theta + 4 = 25(1-\sin^2\theta)$		M1	1.1b
	$50\cos^2\theta + 20\cos\theta - 21 = 0$	$50\sin^2\theta - 20\sin\theta - 21 = 0$		
	$\cos\theta = \frac{-20 \pm \sqrt{4600}}{100}$, o.e.	$\sin\theta = \frac{20 \pm \sqrt{4600}}{100}$, o.e.	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arccos\left(\frac{-2+\sqrt{46}}{10}\right)$	e.g. $\theta = \arcsin\left(\frac{2+\sqrt{46}}{10}\right)$	dM1	1.1b
	$\theta =$ awrt $61.4°$, awrt $208.6°$		A1	2.1
			(5)	

Notes for Question 7

(i)	
B1:	For recalling that $\sec x = \frac{1}{\cos x}$
M1:	Correct strategy of • **Way 1:** applying $\sin 2x = 2\sin x\cos x$ and proceeding to $\sin 2x = k,\ \lvert k\rvert \le 1,\ k \ne 0$ • **Way 2:** squaring both sides, applying $\cos^2 x + \sin^2 x = 1$ and solving a quadratic equation in either $\sin^2 x$ or $\cos^2 x$ to give $\sin^2 x = k$ or $\cos^2 x = k,\ \lvert k\rvert \le 1,\ k \ne 0$
dM1:	Uses the correct order of operations to find at least one value for x in either radians or degrees
A1:	Clear reasoning to achieve both $x = \frac{\pi}{12}, \frac{5\pi}{12}$ and no other values in the range $0 \le x < \frac{\pi}{2}$
Note:	Give dM1 for $\sin 2x = \frac{1}{2} \Rightarrow$ any of $\frac{\pi}{12}, \frac{5\pi}{12}$, 15°, 75°, awrt 0.26 or awrt 1.3
Note:	Give special case, SC B1M0M0A0 for writing down any of $\frac{\pi}{12}, \frac{5\pi}{12}$, 15° or 75° with no working

Notes for Question 7 Continued	
(ii)	
M1:	See scheme
Note:	**Alternative strategy:** Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\cos(\theta+\alpha) = -2,$ finds both R and α, and proceeds to $\cos(\theta+\alpha) = k$, $\lvert k\rvert < 1$, $k \neq 0$
M1:	Either • uses $R\sin(\theta-\alpha)$ to find the values of both R and α • attempts to apply $(5\sin\theta - 5\cos\theta)^2 = 2^2$, uses $\cos^2\theta + \sin^2\theta = 1$ and proceeds to find an equation of the form $\pm\lambda \pm \mu\sin 2\theta = \pm\beta$ or $\pm\mu\sin 2\theta = \pm\beta$; $\mu \neq 0$ • attempts to apply $(5\sin\theta)^2 = (2+5\cos\theta)^2$ or $(5\sin\theta - 2)^2 = (5\cos\theta)^2$ and uses $\cos^2\theta + \sin^2\theta = 1$ to form an equation in $\cos\theta$ only or $\sin\theta$ only
A1:	For $\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$, o.e., $\cos(\theta + 45°) = -\frac{2}{\sqrt{50}}$, o.e. or $\sin 2\theta = \frac{21}{25}$, o.e. or $\cos\theta = \frac{-20 \pm \sqrt{4600}}{100}$, o.e. or $\cos\theta =$ awrt 0.48, awrt -0.88 or $\sin\theta = \frac{20 \pm \sqrt{4600}}{100}$, o.e., or $\sin\theta =$ awrt 0.88, awrt -0.48
Note:	$\sin(\theta - 45°)$, $\cos(\theta + 45°)$, $\sin 2\theta$ must be made the subject for A1
dM1:	**dependent on the first M mark** Uses the correct order of operations to find at least one value for x in either degrees or radians
Note:	dM1 can also be given for $\theta = 180° - \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$ or $\theta = \frac{1}{2}\left(180° - \arcsin\left(\frac{21}{25}\right)\right)$
A1:	Clear reasoning to achieve both $\theta =$ awrt 61.4°, awrt 208.6° and no other values in the range $0 \leq \theta < 360°$
Note:	Give M0M0A0M0A0 for writing down any of $\theta =$ awrt 61.4°, awrt 208.6° with no working
Note:	**<u>Alternative solutions:</u>** (to be marked in the same way as Alt 1): • $5\sin\theta - 5\cos\theta = 2 \Rightarrow 5\tan\theta - 5 = 2\sec\theta \Rightarrow (5\tan\theta - 5)^2 = (2\sec\theta)^2$ $\Rightarrow 25\tan^2\theta - 50\tan\theta + 25 = 4\sec^2\theta \Rightarrow 25\tan^2\theta - 50\tan\theta + 25 = 4(1+\tan^2\theta)$ $\Rightarrow 21\tan^2\theta - 50\tan\theta + 21 = 0 \Rightarrow \tan\theta = \frac{50 \pm \sqrt{736}}{42} = \frac{25 \pm 2\sqrt{46}}{21} = 1.8364..., 0.5445...$ $\Rightarrow \theta =$ awrt 61.4°, awrt 208.6° only • $5\sin\theta - 5\cos\theta = 2 \Rightarrow 5 - 5\cot\theta = 2\operatorname{cosec}\theta \Rightarrow (5 - 5\cot\theta)^2 = (2\operatorname{cosec}\theta)^2$ $\Rightarrow 25 - 50\cot\theta + 25\cot^2\theta = 4\operatorname{cosec}^2\theta \Rightarrow 25 - 50\cot\theta + 25\cot^2\theta = 4(1+\cot^2\theta)$ $\Rightarrow 21\cot^2\theta - 50\cot\theta + 21 = 0 \Rightarrow \cot\theta = \frac{50 \pm \sqrt{736}}{42} = \frac{25 \pm 2\sqrt{46}}{21} = 1.8364..., 0.5445...$ $\Rightarrow \theta =$ awrt 61.4°, awrt 208.6° only

Question	Scheme	Marks	AOs
8 (a) **Way 1**	$H = Ax(40-x)$ {or $H = Ax(x-40)$ }	M1	3.3
	$x = 20,\ H = 12 \Rightarrow 12 = A(20)(40-20) \Rightarrow A = \dfrac{3}{100}$	dM1	3.1b
	$H = \dfrac{3}{100}x(40-x)$ or $H = -\dfrac{3}{100}x(x-40)$	A1	1.1b
		(3)	
(a) **Way 2**	$H = 12 - \lambda(x-20)^2$ {or $H = 12 + \lambda(x-20)^2$ }	M1	3.3
	$x = 40,\ H = 0 \Rightarrow 0 = 12 - \lambda(40-20)^2 \Rightarrow \lambda = \dfrac{3}{100}$	dM1	3.1b
	$H = 12 - \dfrac{3}{100}(x-20)^2$	A1	1.1b
		(3)	
(a) **Way 3**	$H = ax^2 + bx + c$ (or deduces $H = ax^2 + bx$) **Both** $x = 0,\ H = 0 \Rightarrow 0 = 0 + 0 + c \Rightarrow c = 0$ **and either** $x = 40,\ H = 0 \Rightarrow 0 = 1600a + 40b$ **or** $x = 20,\ H = 12 \Rightarrow 12 = 400a + 20b$ **or** $\dfrac{-b}{2a} = 20$ $\{\Rightarrow b = -40a\}$	M1	3.3
	$b = -40a \Rightarrow 12 = 400a + 20(-40a) \Rightarrow a = -0.03$ so $b = -40(-0.03) = 1.2$	dM1	3.1b
	$H = -0.03x^2 + 1.2x$	A1	1.1b
		(3)	
(b)	$\{H = 3 \Rightarrow\}\ 3 = \dfrac{3}{100}x(40-x) \Rightarrow x^2 - 40x + 100 = 0$ **or** $\{H = 3 \Rightarrow\}\ 3 = 12 - \dfrac{3}{100}(x-20)^2 \Rightarrow (x-20)^2 = 300$	M1	3.4
	e.g. $x = \dfrac{40 \pm \sqrt{1600 - 4(1)(100)}}{2(1)}$ or $x = 20 \pm \sqrt{300}$	dM1	1.1b
	$\left\{\text{chooses } 20 + \sqrt{300} \Rightarrow\right\}$ greatest distance $=$ awrt 37.3 m	A1	3.2a
		(3)	
(c)	Gives a limitation of the model. Accept e.g. • the ground is horizontal • the ball needs to be kicked from the ground • the ball is modelled as a particle • the horizontal bar needs to be modelled as a line • there is no wind or air resistance on the ball • there is no spin on the ball • no obstacles in the trajectory (or path) of the ball • the trajectory of the ball is a perfect parabola	B1	3.5b
		(1)	
			(7 marks)

Notes for Question 8	
(a)	
M1:	Translates the situation given into a suitable equation for the model. E.g. **Way 1:** {Uses $(0, 0)$ and $(40, 0)$ to write} $H = Ax(40-x)$ o.e. {or $H = Ax(x-40)$ }
	Way 2: {Uses $(20, 12)$ to write} $H = 12 - \lambda(x-20)^2$ or $H = 12 + \lambda(x-20)^2$
	Way 3: Writes $H = ax^2 + bx + c$, **and** uses $(0, 0)$ to deduce $c = 0$ **and** an attempt at using either $(40, 0)$ or $(20, 12)$ **Special Case:** Allow SC M1dM0A0 for not deducing $c = 0$ but attempting to apply both $(40, 0)$ and $(20, 12)$
dM1:	Applies a complete strategy with appropriate constraints to find all constants in their model. **Way 1:** Uses $(20, 12)$ on their model and finds $A = ...$ **Way 2:** Uses either $(40, 0)$ or $(0, 0)$ on their model to find $\lambda = ...$ **Way 3:** Uses $(40, 0)$ and $(20, 12)$ on their model to find $a = ...$ and $b = ...$
A1:	Finds a correct equation linking *H* to *x* E.g. $H = \frac{3}{100}x(40-x)$, $H = 12 - \frac{3}{100}(x-20)^2$ or $H = -0.03x^2 + 1.2x$
Note:	Condone writing *y* in place of *H* for the M1 and dM1 marks.
Note:	Give final A0 for $y = -0.03x^2 + 1.2x$
Note:	Give special case M1dM0A0 for writing down any of $H = 12 - (x-20)^2$ or $H = x(40-x)$ or $H = x(x-40)$
Note:	Give M1 dM1 for finding $-0.03x^2 + 1.2x$ or $a = -0.03, b = 1.2, c = 0$ in an implied $ax^2 + bx$ or $ax^2 + bx + c$ (with no indication of $H = ...$)
(b)	
M1:	Substitutes $H = 3$ into their quadratic equation and proceeds to obtain a 3TQ or a quadratic in the form $(x \pm \alpha)^2 = \beta;\ \alpha, \beta \neq 0$
Note:	E.g. $1.2x - 0.03x^2 = 3$ or $40x - x^2 = 100$ are acceptable for the 1st M mark
Note:	Give M0 dM0 A0 for $(\text{their } A)x^2 = 3 \Rightarrow x = ...$ or their $(\text{their } A)x^2 + (\text{their } k) = 3 \Rightarrow x = ...$
dM1:	Correct method of solving their quadratic equation to give ***at least one solution***
A1:	Interprets their solution in the original context by selecting the larger correct value ***and states correct units for their value***. E.g. Accept awrt 37.3 m or $\left(20 + \sqrt{300}\right)$ m or $\left(20 + 10\sqrt{3}\right)$ m
Note:	Condone the use of inequalities for the method marks in part (b)
(c):	
B1:	See scheme
Note:	Give no credit for the following reasons • *H* (or the height of ball) is negative when $x > 40$ • Bounce of the ball should be considered after hitting the ground • Model will not be true for a different rugby ball • Ball may not be kicked in the same way each time

Question	Scheme	Marks	AOs
9	$\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$; as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$		
	$\frac{\cos(\theta+h)-\cos\theta}{h}$	B1	2.1
	$= \frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$	M1	1.1b
		A1	1.1b
	$= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$		
	As $h \to 0$, $-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta \to -1\sin\theta + 0\cos\theta$	dM1	2.1
	so $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$ *	A1*	2.5
		(5)	
		(5 marks)	

Notes for Question 9

B1:	Gives the correct fraction such as $\frac{\cos(\theta+h)-\cos\theta}{h}$ or $\frac{\cos(\theta+\delta\theta)-\cos\theta}{\delta\theta}$ Allow $\frac{\cos(\theta+h)-\cos\theta}{(\theta+h)-\theta}$ o.e. **Note:** $\cos(\theta+h)$ or $\cos(\theta+\delta\theta)$ may be expanded
M1:	Uses the compound angle formula for $\cos(\theta+h)$ to give $\cos\theta\cos h \pm \sin\theta\sin h$
A1:	Achieves $\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$ or equivalent
dM1:	**dependent on both the B and M marks being awarded** Complete attempt to apply the given limits to the gradient of their chord
Note:	They must isolate $\frac{\sin h}{h}$ and $\left(\frac{\cos h - 1}{h}\right)$, and replace $\frac{\sin h}{h}$ with 1 and replace $\left(\frac{\cos h - 1}{h}\right)$ with 0
A1*:	cso. Uses correct mathematical language of limiting arguments to prove $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$
Note:	Acceptable responses for the final A mark include: • $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = \lim_{h\to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right) = -1\sin\theta + 0\cos\theta = -\sin\theta$ • Gradient of chord $= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of chord tends to the gradient of the curve, so derivative is $-\sin\theta$ • Gradient of chord $= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of ***curve*** is $-\sin\theta$
Note:	Give final A0 for the following example which shows ***no limiting arguments***: when $h = 0$, $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta = -1\sin\theta + 0\cos\theta = -\sin\theta$
Note:	Do not allow the final A1 for stating $\frac{\sin h}{h} = 1$ or $\left(\frac{\cos h - 1}{h}\right) = 0$ and attempting to apply these
Note:	In this question $\delta\theta$ may be used in place of h
Note:	Condone $\mathrm{f}'(\theta)$ where $\mathrm{f}(\theta) = \cos\theta$ or $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ where $y = \cos\theta$ used in place of $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta)$

Notes for Question 9 Continued	
Note:	Condone x used in place of θ if this is done consistently
Note:	Give final A0 for • $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos x) = \lim_{h \to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right) = -1\sin\theta + 0\cos\theta = -\sin\theta$ • $\frac{\mathrm{d}}{\mathrm{d}\theta} = \ldots$ • Defining $\mathrm{f}(x) = \cos\theta$ and applying $\mathrm{f}'(x) = \ldots$ • $\frac{\mathrm{d}}{\mathrm{d}x}(\cos\theta)$
Note:	Give final A1 for a correct limiting argument in x, followed by $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$ e.g. $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos x) = \lim_{h \to 0}\left(-\frac{\sin h}{h}\sin x + \left(\frac{\cos h - 1}{h}\right)\cos x\right) = -1\sin x + 0\cos x = -\sin x$ $\Rightarrow \frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$
Note:	Applying $h \to 0,\ \sin h \to h,\ \cos h \to 1$ to give e.g. $\lim_{h \to 0}\left(\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}\right) = \left(\frac{\cos\theta(1) - \sin\theta(h) - \cos\theta}{h}\right) = \frac{-\sin\theta(h)}{h} = -\sin\theta$ is final M0 A0 for incorrect application of limits
Note:	$\lim_{h \to 0}\left(\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}\right) = \lim_{h \to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right)$ $= \underleftrightarrow{\lim_{h \to 0}\ (-(1)\sin\theta + 0\cos\theta)} = -\sin\theta$. So for not removing $\underleftrightarrow{\lim_{h \to 0}}$ when the limit was taken is final A0
Note:	**<u>Alternative Method:</u>** Considers $\frac{\cos(\theta + h) - \cos(\theta - h)}{(\theta + h) - (\theta - h)}$ which simplifies to $\frac{-2\sin\theta\sin h}{2h}$

Question	Scheme	Marks	AOs
10 (a)	$\frac{\mathrm{d}r}{\mathrm{d}t} \propto \pm\frac{1}{r^2}$ or $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{k}{r^2}$ (for k or a numerical k)	M1	3.3
	$\int r^2\,\mathrm{d}r = \int \pm k\,\mathrm{d}t \Rightarrow ...$ (for k or a numerical k)	M1	2.1
	$\frac{1}{3}r^3 = \pm kt\ \{+c\}$	A1	1.1b
	$t=0,\ r=5$ and $t=4,\ r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$, where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth OR $t=0,\ r=5$ and $t=240,\ r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$, where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth	M1 A1	3.1a 1.1b
		(5)	
(b)	$r = 0 \Rightarrow 0 = -\frac{49}{6}t + \frac{125}{3} \Rightarrow 0 = -49t + 250 \Rightarrow t = ...$	M1	3.4
	time = 5 minutes 6 seconds	A1	1.1b
		(2)	
(c)	Suggests a suitable limitation of the model. E.g. • Model does not consider how the mint is sucked • Model does not consider whether the mint is bitten • Model is limited for times up to 5 minutes 6 seconds, o.e. • Not valid for times greater than 5 minutes 6 seconds, o.e. • Mint may not retain the shape of a sphere (or have uniform radius) as it is being sucked • The model indicates that the radius of the mint is negative after it dissolves • Model does not consider the temperature in the mouth • Model does not consider rate of saliva production • Mint could be swallowed before it dissolves in the mouth	B1	3.5b
		(1)	
			(8 marks)

Notes for Question 10	
(a)	
M1:	Translates the description of the model into mathematics. See scheme.
M1:	Separates the variables of their differential equation which is in the form $\frac{\mathrm{d}r}{\mathrm{d}t} = \mathrm{f}(r)$ and some attempt at integration. (e.g. attempts to integrate at least one side). e.g. $\int r^2\,\mathrm{d}r = \int \pm k\,\mathrm{d}t$ and some attempt at integration. Condone the lack of integral signs
Note:	You can imply the M1 mark for $r^2 dr = -k\,\mathrm{d}t \Rightarrow \frac{1}{3}r^3 = -kt$
Note:	A numerical value of k (e.g. $k = \pm 1$) is allowed for the first two M marks
A1:	Correct integration to give $\frac{1}{3}r^3 = \pm kt$ with or without a constant of integration, c
M1:	For a complete process of using the boundary conditions to find both their unknown constants and finds an equation linking r and t So applies either • $t=0,\ r=5$ and $t=4,\ r=3$, or • $t=0,\ r=5$ and $t=240,\ r=3$, ***on their integrated equation*** to find their constants k and c ***and*** obtains an equation linking r and t
A1:	Correct equation, with variables r and t fully defined including correct reference to units. • $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$, {or an equivalent equation,} where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth • $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$, {or an equivalent equation,} where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth
Note:	Allow correct equations such as • in minutes, $r = \sqrt[3]{\frac{250-49t}{2}}$, $r^3 = -\frac{49}{2}t + 125$ or $t = \frac{250-2r^3}{49}$ • in seconds, $r = \sqrt[3]{\frac{15000-49t}{120}}$, $r^3 = -\frac{49}{120}t + 125$ or $t = \frac{15000-120r^3}{49}$
Note:	t defined as "the time from the start" is not sufficient for the final A1
(b)	
M1:	Sets $r = 0$ in their part (a) equation which links r with t and rearranges to make $t = \ldots$
A1:	5 minutes 6 seconds cao (**Note:** 306 seconds with no reference to 5 minutes 6 seconds is A0)
Note:	Give M0 if their equation would solve to give a negative time or a negative time is found
Note:	You can mark part (a) and part (b) together
(c)	
B1:	See scheme
Note:	Do not accept by itself • mint may not dissolve at a constant rate • rate of decrease of mint must be constant • $0 \le t < \frac{250}{49}$, $r \ge 0$; without any written explanation • reference to a mint having $r > 5$

Question	Scheme	Marks	AOs
11	$\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A+\dfrac{B}{(x-3)}+\dfrac{C}{(1-2x)}$		
(a) Way 1	$1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using a correct identity	A1	1.1b
		(4)	
(a) Way 2	{long division gives} $\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3+\dfrac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using $-10x+10 \equiv B(1-2x)+C(x-3)$	A1	1.1b
		(4)	
(b)	$f(x)=3+\dfrac{4}{(x-3)}-\dfrac{2}{(1-2x)}$ $\{=3+4(x-3)^{-1}-2(1-2x)^{-1}\}$; $x>3$		
	$f'(x)=-4(x-3)^{-2}-4(1-2x)^{-2}$ $\left\{=-\dfrac{4}{(x-3)^2}-\dfrac{4}{(1-2x)^2}\right\}$	M1 A1ft	2.1 1.1b
	Correct $f'(x)$ **and** as $(x-3)^2>0$ and $(1-2x)^2>0$, then $f'(x)=-(+\text{ve})-(+\text{ve})<0$, so $f(x)$ is a decreasing function	A1	2.4
		(3)	
		(7 marks)	

Notes for Question 11

(a)	
M1:	**Way 1:** Uses a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ in a complete method to find values for *B* and *C*. **Note:** Allow one slip in copying $1+11x-6x^2$ **Way 2:** Uses a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ (which has been found from long division) in a complete method to find values for *B* and *C*
B1:	$A=3$
M1:	Attempts to find the value of either *B* or *C* from their identity This can be achieved by ***either*** substituting values into their identity ***or*** by comparing coefficients and solving the resulting equations simultaneously
A1:	See scheme
Note:	**Way 1:** Comparing terms: $x^2: -6=-2A;\quad x: 11=7A-2B+C;\quad \text{constant}: 1=-3A+B-3C$ **Way 1:** Substituting: $x=3: -20=-5B \Rightarrow B=4$; $x=\frac{1}{2}: 5=-\frac{5}{2}C \Rightarrow C=-2$
Note:	**Way 2:** Comparing terms: $x: -10=-2B+C;\quad \text{constant}: 10=B-3C$ **Way 2:** Substituting: $x=3: -20=-5B \Rightarrow B=4$; $x=\frac{1}{2}: 5=-\frac{5}{2}C \Rightarrow C=-2$

Note:	$A=3,\ B=4,\ C=-2$ from no working scores M1B1M1A1
Note:	The final A1 mark is effectively dependent upon both M marks

Notes for Question 11 Continued	
(a) ctd	
Note:	Writing $1+11x-6x^2 \equiv B(1-2x)+C(x-3) \Rightarrow B=4,\ C=-2$ will get 1st M0, 2nd M1, 1st A0
Note:	**Way 1:** You can imply a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ from seeing $\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \dfrac{A(1-2x)(x-3)+B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
Note:	**Way 2:** You can imply a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ from seeing $\dfrac{-10x+10}{(x-3)(1-2x)} \equiv \dfrac{B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
(b)	
M1:	Differentiates to give $\{f'(x)=\}\ \pm\lambda(x-3)^{-2} \pm \mu(1-2x)^{-2};\ \lambda,\ \mu \neq 0$
A1ft:	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$, which can be simplified or un-simplified
Note:	Allow A1ft for $f'(x) = -(\text{their } B)(x-3)^{-2} + (2)(\text{their } C)(1-2x)^{-2}$; $(\text{their } B), (\text{their } C) \neq 0$
A1:	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$ or $f'(x) = -\dfrac{4}{(x-3)^2} - \dfrac{4}{(1-2x)^2}$ **and** a correct explanation e.g. $f'(x) = -(+\text{ ve}) - (+\text{ ve}) < 0$, so $f(x)$ is a decreasing {function}
Note:	The final A mark can be scored in part (b) from an incorrect $A = \ldots$ or from $A=0$ or no value of A found in part (a)

Notes for Question 11 Continued - Alternatives

(a)

Note: Be aware of the following alternative solutions, by initially dividing by "$(x-3)$" or "$(1-2x)$"

- $$\frac{1+11x-6x^2}{"(x-3)"(1-2x)} \equiv \frac{-6x-7}{(1-2x)} - \frac{20}{(x-3)(1-2x)} \equiv 3 - \frac{10}{(1-2x)} - \frac{20}{(x-3)(1-2x)}$$

  $$\frac{20}{(x-3)(1-2x)} \equiv \frac{D}{(x-3)} + \frac{E}{(1-2x)} \Rightarrow 20 \equiv D(1-2x) + E(x-3) \Rightarrow D = -4,\ E = -8$$

  $$\Rightarrow 3 - \frac{10}{(1-2x)} - \left(\frac{-4}{(x-3)} + \frac{-8}{(1-2x)}\right) \equiv 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)};\ A=3,\ B=4,\ C=-2$$

- $$\frac{1+11x-6x^2}{(x-3)"(1-2x)"} \equiv \frac{3x-4}{(x-3)} + \frac{5}{(x-3)(1-2x)} \equiv 3 + \frac{5}{(x-3)} + \frac{5}{(x-3)(1-2x)}$$

  $$\frac{5}{(x-3)(1-2x)} \equiv \frac{D}{(x-3)} + \frac{E}{(1-2x)} \Rightarrow 5 \equiv D(1-2x) + E(x-3) \Rightarrow D = -1,\ E = -2$$

  $$\Rightarrow 3 + \frac{5}{(x-3)} + \left(\frac{-1}{(x-3)} + \frac{-2}{(1-2x)}\right) \equiv 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)};\ A=3,\ B=4,\ C=-2$$

(b)

Alternative Method 1:

$$\text{f}(x) = \frac{1+11x-6x^2}{(x-3)(1-2x)},\ x>3 \Rightarrow \text{f}(x) = \frac{1+11x-6x^2}{-2x^2+7x-3};\ \begin{cases} u = 1+11x-6x^2 & v = -2x^2+7x-3 \\ u' = 11-12x & v' = -4x+7 \end{cases}$$

$\text{f}'(x) = \dfrac{(-2x^2+7x-3)(11-12x)-(1+11x-6x^2)(-4x+7)}{(-2x^2+7x-3)^2}$	Uses quotient rule to find $\text{f}'(x)$	**M1**
	Correct differentiation	**A1**
$\text{f}'(x) = \dfrac{-20((x-1)^2+1)}{(-2x^2+7x-3)^2}$ and a correct explanation, e.g. $\text{f}'(x) = -\dfrac{(+\text{ ve})}{(+\text{ ve})} < 0$, so $\text{f}(x)$ is a decreasing {function}		**A1**

Alternative Method 2:

Allow M1A1A1 for the following solution:

Given $\text{f}(x) = 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)} = 3 + \dfrac{4}{(x-3)} + \dfrac{2}{(2x-1)}$

as $\dfrac{4}{(x-3)}$ decreases when $x > 3$ and $\dfrac{2}{(2x-1)}$ decreases when $x > 3$

then $\text{f}(x)$ is a decreasing {function}

Question	Scheme	Marks	AOs
12	$1-\cos 2\theta \equiv \tan\theta \sin 2\theta,\ \theta \neq \dfrac{(2n+1)\pi}{2},\ n \in \mathbb{Z}$		
(a) Way 1	$\tan\theta\sin 2\theta = \left(\dfrac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta)$	M1	1.1b
	$= \left(\dfrac{\sin\theta}{\cancel{\cos\theta}}\right)(2\sin\theta\cancel{\cos\theta}) = 2\sin^2\theta = 1-\cos 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
(a) Way 2	$1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta$	M1	1.1b
	$= \left(\dfrac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta) = \tan\theta\sin 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
	$(\sec^2 x-5)(1-\cos 2x) = 3\tan^2 x\sin 2x,\ -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$		
(b) Way 1	$(\sec^2 x-5)\tan x\sin 2x = 3\tan^2 x\sin 2x$ **or** $(\sec^2 x-5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$		
	Deduces $x=0$	B1	2.2a
	Uses $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ e.g. $(1+\tan^2 x-3\tan x-5)\tan x=0$ **or** $(1+\tan^2 x-3\tan x-5)(1-\cos 2x)=0$ **or** $1+\tan^2 x-5=3\tan x$	M1	2.1
	$\tan^2 x-3\tan x-4=0$	A1	1.1b
	$(\tan x-4)(\tan x+1)=0 \Rightarrow \tan x = ...$	M1	1.1b
	$x = -\dfrac{\pi}{4},\ 1.326$	A1	1.1b
		A1	1.1b
		(6)	
			(9 marks)

Notes for Question 12

(a)	**Way 1**
M1:	Applies $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to $\tan\theta\sin 2\theta$
M1:	Cancels as scheme (may be implied) and attempts to use $\cos 2\theta = 1-2\sin^2\theta$
A1*:	For a correct proof showing all steps of the argument
(a) Way 2	
M1:	For using $\cos 2\theta = 1-2\sin^2\theta$
Note:	If the form $\cos 2\theta = \cos^2\theta-\sin^2\theta$ or $\cos 2\theta = 2\cos^2\theta-1$ is used, the mark cannot be awarded until $\cos^2\theta$ has been replaced by $1-\sin^2\theta$
M1:	Attempts to write their $2\sin^2\theta$ in terms of $\tan\theta$ and $\sin 2\theta$ using $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and
	$\sin 2\theta = 2\sin\theta\cos\theta$ within the given expression
A1*:	For a correct proof showing all steps of the argument
Note:	If a proof meets in the middle; e.g. they show LHS = $2\sin^2\theta$ and RHS = $2\sin^2\theta$; then some indication must be given that the proof is complete. E.g. $1-\cos 2\theta \equiv \tan\theta\sin 2\theta$, QED, box

Notes for Question 12 Continued	
(b)	
B1:	Deduces that the given equation yields a solution $x = 0$
M1:	For using the key step of $\sec^2 x = 1 + \tan^2 x$ and cancels/factorises out $\tan x$ or $(1 - \cos 2x)$ or $\sin 2x$ to produce a quadratic factor or quadratic equation in just $\tan x$
Note:	Allow the use of $\pm\sec^2 x = \pm 1 \pm \tan^2 x$ for M1
A1:	Correct 3TQ in $\tan x$. E.g. $\tan^2 x - 3\tan x - 4 = 0$
Note:	E.g. $\tan^2 x - 4 = 3\tan x$ or $\tan^2 x - 3\tan x = 4$ are acceptable for A1
M1:	For a correct method of solving their 3TQ in $\tan x$
A1:	Any one of $-\frac{\pi}{4}$, awrt -0.785, awrt 1.326, $-45°$, awrt $75.964°$
A1:	Only $x = -\frac{\pi}{4}, 1.326$ **cao** stated in the range $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Note: <u>**Alternative Method (Alt 1)**</u>

$(\sec^2 x - 5)\tan x \sin 2x = 3\tan^2 x \sin 2x$ **or** $(\sec^2 x - 5)(1 - \cos 2x) = 3\tan x(1 - \cos 2x)$			
Deduces $x = 0$		B1	2.2a
$\sec^2 x - 5 = 3\tan x \Rightarrow \frac{1}{\cos^2 x} - 5 = 3\left(\frac{\sin x}{\cos x}\right)$ $1 - 5\cos^2 x = 3\sin x \cos x$ $1 - 5\left(\frac{1 + \cos 2x}{2}\right) = \frac{3}{2}\sin 2x$	Complete process (as shown) of using the identities for $\sin 2x$ and $\cos 2x$ to proceed as far as $\pm A \pm B\cos 2x = \pm C\sin 2x$	M1	2.1
$-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ $\{3\sin 2x + 5\cos 2x = -3\}$	$-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ o.e.	A1	1.1b
$\sqrt{34}\sin(2x + 1.03) = -3$	Expresses their answer in the form $R\sin(2x + \alpha) = k$; $k \neq 0$ with values for R and α	M1	1.1b
$\sin(2x + 1.03) = -\frac{3}{\sqrt{34}}$			
$x = -\frac{\pi}{4}, 1.326$		A1 A1	1.1b 1.1b

Question	Scheme	Marks	AOs
13	$C: y = x\ln x$; l is a normal to C at $P(\text{e}, \text{e})$ Let x_A be the x-coordinate of where l cuts the x-axis		
	$\dfrac{\text{d}y}{\text{d}x} = \ln x + x\left(\dfrac{1}{x}\right) \quad \{= 1 + \ln x\}$	M1 A1	2.1 1.1b
	$x = \text{e},\ m_T = 2 \Rightarrow m_N = -\dfrac{1}{2} \Rightarrow y - \text{e} = -\dfrac{1}{2}(x - \text{e})$ $y = 0 \Rightarrow -\text{e} = -\dfrac{1}{2}(x - \text{e}) \Rightarrow x = ...$	M1	3.1a
	l meets x-axis at $x = 3\text{e}$ (allow $x = 2\text{e} + \text{e}\ln\text{e}$)	A1	1.1b
	{Areas:} **either** $\int_1^{\text{e}} x\ln x\,\text{d}x = [\ ...\ ]_1^{\text{e}} = ...$ **or** $\dfrac{1}{2}((\text{their } x_A) - \text{e})\text{e}$	M1	2.1
	$\left\{\int x\ln x\,\text{d}x = \right\}\ \dfrac{1}{2}x^2\ln x - \int \dfrac{1}{x}.\left(\dfrac{x^2}{2}\right)\{\text{d}x\}$	M1	2.1
	$\left\{= \dfrac{1}{2}x^2\ln x - \int \dfrac{1}{2}x\,\{\text{d}x\}\right\} = \dfrac{1}{2}x^2\ln x - \dfrac{1}{4}x^2$	dM1 A1	1.1b 1.1b
	$\text{Area}(R_1) = \int_1^{\text{e}} x\ln x\,\text{d}x = [\ ...\ ]_1^{\text{e}} = ...;\ \text{Area}(R_2) = \dfrac{1}{2}((\text{their } x_A) - \text{e})\text{e}$ and so, $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2) \quad \{= \frac{1}{4}\text{e}^2 + \frac{1}{4} + \text{e}^2\}$	M1	3.1a
	$\text{Area}(R) = \frac{5}{4}\text{e}^2 + \frac{1}{4}$	A1	1.1b
		(10)	

Notes for Question 13

M1:	Differentiates by using the product rule to give $\ln x + x(\text{their } \text{g}'(x))$, where $\text{g}(x) = \ln x$
A1:	Correct differentiation of $y = x\ln x$, which can be un-simplified or simplified
M1:	Complete strategy to find the x coordinate where their normal to C at $P(\text{e}, \text{e})$ meets the x-axis i.e. Sets $y = 0$ in $y - \text{e} = m_N(x - \text{e})$ to find $x = ...$
Note:	m_T is found by using calculus and $m_N \neq m_T$
A1:	l meets x-axis at $x = 3\text{e}$, allowing un-simplified values for x such as $x = 2\text{e} + \text{e}\ln\text{e}$
Note:	Allow $x = \text{awrt } 8.15$
M1:	Scored for either • Area under curve $= \int_1^{\text{e}} x\ln x\,\text{d}x = [\ ...\ ]_1^{\text{e}} = ...$, with limits of e and 1 and some attempt to substitute these and subtract • **or** Area under line $= \dfrac{1}{2}((\text{their } x_A) - \text{e})\text{e}$, with a valid attempt to find x_A
M1:	Integration by parts the correct way around to give $Ax^2\ln x - \int B\left(\dfrac{x^2}{x}\right)\{\text{d}x\}$; $A \neq 0,\ B > 0$
dM1:	**dependent on the previous M mark** Integrates the second term to give $\pm\lambda x^2$; $\lambda \neq 0$
A1:	$\dfrac{1}{2}x^2\ln x - \dfrac{1}{4}x^2$
M1:	Complete strategy of finding the area of R by finding the sum of two key areas. See scheme.
A1:	$\frac{5}{4}\text{e}^2 + \frac{1}{4}$

Notes for Question 13 Continued	
Note:	$\text{Area}(R_2)$ can also be found by integrating the line *l* between limits of e and their x_A i.e. $\text{Area}(R_2) = \int_{\text{e}}^{\text{their } x_A} \left(-\frac{1}{2}x + \frac{3}{2}\text{e}\right) \text{d}x = \left[\ \ldots \ \right]_{\text{e}}^{\text{their } x_A} = \ldots$
Note:	**<u>Calculator approach with no algebra, differentiation or integration seen:</u>** • Finding *l* cuts through the *x*-axis at awrt 8.15 is 2nd M1 2nd A1 • Finding area between curve and the *x*-axis between $x = 1$ and $x = \text{e}$ to give awrt 2.10 is 3rd M1 • Using the above information (must be seen) to apply $\text{Area}(R) = 2.0972... + 7.3890... = 9.4862...$ is final M1 Therefore, a maximum of 4 marks out of the 10 available.

Question	Scheme	Marks	AOs
14	$N=\dfrac{900}{3+7e^{-0.25t}}=900(3+7e^{-0.25t})^{-1},\ t\in\mathbb{R},\ t\geq 0\ ;\quad \dfrac{dN}{dt}=\dfrac{N(300-N)}{1200}$		
(a)	90	B1	3.4
		(1)	
(b) Way 1	$\dfrac{dN}{dt}=-900(3+7e^{-0.25t})^{-2}\left(7(-0.25)e^{-0.25t}\right)\quad\left\{=\dfrac{900(0.25)(7)e^{-0.25t}}{(3+7e^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\Rightarrow \dfrac{dN}{dt}=\dfrac{900(0.25)\left(\left(\dfrac{900}{N}-3\right)\right)}{\left(\dfrac{900}{N}\right)^2}$	dM1	2.1
	correct algebra leading to $\dfrac{dN}{dt}=\dfrac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	
(b) Way 2	$\dfrac{dN}{dt}=-900(3+7e^{-0.25t})^{-2}\left(7(-0.25)e^{-0.25t}\right)\quad\left\{=\dfrac{900(0.25)(7)e^{-0.25t}}{(3+7e^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\dfrac{N(300-N)}{1200}=\dfrac{\left(\dfrac{900}{3+7e^{-0.25t}}\right)\left(300-\dfrac{900}{3+7e^{-0.25t}}\right)}{1200}$	dM1	2.1
	LHS $=\dfrac{1575e^{-0..25t}}{(3+7e^{-0.25t})^2}$ o.e., RHS $=\dfrac{900(300(3+7e^{-0.25t})-900)}{1200(3+7e^{-0.25t})^2}=\dfrac{1575e^{-0..25t}}{(3+7e^{-0.25t})^2}$ o.e. and states hence $\dfrac{dN}{dt}=\dfrac{N(300-N)}{1200}$ (or LHS = RHS) *	A1*	1.1b
		(4)	
(c)	Deduces $N=150$ (can be implied)	B1	2.2a
	so $150=\dfrac{900}{3+7e^{-0.25T}}\Rightarrow e^{-0.25T}=\dfrac{3}{7}$	M1	3.4
	$T=-4\ln\left(\dfrac{3}{7}\right)$ or $T=$ awrt 3.4 (months)	dM1	1.1b
		A1	1.1b
		(4)	
(d)	either one of 299 or 300	B1	3.4
		(1)	
			(10 marks)

Notes for Question 14	
14 (b)	
M1:	Attempts to differentiate using • the chain rule to give $\frac{dN}{dt} = \pm Ae^{-0.25t}(3+7e^{-0.25t})^{-2}$ or $\frac{\pm Ae^{-0.25t}}{(3+7e^{-0.25t})^2}$ o.e. • the quotient rule to give $\frac{dN}{dt} = \frac{(3+7e^{-0.25t})(0) \pm Ae^{-0.25t}}{(3+7e^{-0.25t})^2}$ • implicit differentiation to give $N(3+7e^{-0.25t}) = 900 \Rightarrow (3+7e^{-0.25t})\frac{dN}{dt} \pm ANe^{-0.25t} = 0$, o.e. where $A \neq 0$
Note:	Condone a slip in copying $(3+7e^{-0.25t})$ for the M mark
A1:	A correct differentiation statement
Note:	Implicit differentiation gives $(3+7e^{-0.25t})\frac{dN}{dt} - 1.75Ne^{-0.25t} = 0$
dM1:	**Way 1:** Complete attempt, by eliminating t, to form an equation linking $\frac{dN}{dt}$ and N only **Way 2:** Complete substitution of $N = \frac{900}{3+7e^{-0.25t}}$ into $\frac{dN}{dt} = \frac{N(300-N)}{1200}$
Note:	**Way 1:** e.g. substitutes $3+7e^{-0.25t} = \frac{900}{N}$ and $e^{-0.25t} = \frac{900}{N}$ or substitutes $e^{-0.25t} = \frac{\frac{900}{N}-3}{7}$ into their $\frac{dN}{dt} = ...$ to form an equation linking $\frac{dN}{dt}$ and N
A1*:	**Way 1:** Correct algebra leading to $\frac{dN}{dt} = \frac{N(300-N)}{1200}$ * **Way 2:** See scheme
(c)	
B1:	Deduces or shows that $\frac{dN}{dt}$ is maximised when $N = 150$
M1:	Uses the model $N = \frac{900}{3+7e^{-0.25t}}$ with their $N = 150$ and proceeds as far as $e^{-0.25T} = k,\ k > 0$ or $e^{0.25T} = k,\ k > 0$. Condone $t \equiv T$
dM1:	Correct method of using logarithms to find a value for T. Condone $t \equiv T$
A1:	see scheme
Note:	$\frac{d^2N}{dt^2} = \frac{dN}{dt}\left(\frac{300}{1200} - \frac{2N}{1200}\right) = 0 \Rightarrow N = 150$ is acceptable for B1
Note:	Ignore units for T
Note:	Applying $300 = \frac{900}{3+7e^{-0.25t}} \Rightarrow t = ...$ or $0 = \frac{900}{3+7e^{-0.25t}} \Rightarrow t = ...$ is M0 dM0 A0
Note:	M1 dM1 can only be gained in (c) by using an N value in the range $90 < N < 300$
(d)	
B1:	300 (or accept 299)

Question	Scheme	Marks	AOs
14	$N=\dfrac{900}{3+7\mathrm{e}^{-0.25t}}=900(3+7\mathrm{e}^{-0.25t})^{-1},\ t\in\mathbb{R},\ t\geq 0\ ;\quad \dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{N(300-N)}{1200}$		
(b) Way 3	$\int \dfrac{1}{N(300-N)}\,\mathrm{d}N=\int \dfrac{1}{1200}\,\mathrm{d}t$	M1	2.1
	$\int \dfrac{1}{300}\left(\dfrac{1}{N}+\dfrac{1}{300-N}\right)\mathrm{d}N=\int \dfrac{1}{1200}\,\mathrm{d}t$ $\dfrac{1}{300}\ln N-\dfrac{1}{300}\ln(300-N)=\dfrac{1}{1200}t\ \{+c\}$	A1	1.1b
	$\{t=0,\ N=90\Rightarrow\}\ \ c=\dfrac{1}{300}\ln(90)-\dfrac{1}{300}\ln(210)\ \Rightarrow c=\dfrac{1}{300}\ln\left(\dfrac{3}{7}\right)$ $\dfrac{1}{300}\ln N-\dfrac{1}{300}\ln(300-N)=\dfrac{1}{1200}t+\dfrac{1}{300}\ln\left(\dfrac{3}{7}\right)$ $\ln N-\ln(300-N)=\dfrac{1}{4}t+\ln\left(\dfrac{3}{7}\right)$ $\ln\left(\dfrac{N}{300-N}\right)=\dfrac{1}{4}t+\ln\left(\dfrac{3}{7}\right)\ \Rightarrow\ \dfrac{N}{300-N}=\dfrac{3}{7}\mathrm{e}^{\frac{1}{4}t}$	dM1	2.1
	$7N=3\mathrm{e}^{\frac{1}{4}t}(300-N)\ \Rightarrow 7N+3N\mathrm{e}^{\frac{1}{4}t}=900\mathrm{e}^{\frac{1}{4}t}$ $N(7+3\mathrm{e}^{\frac{1}{4}t})=900\mathrm{e}^{\frac{1}{4}t}\ \Rightarrow\ N=\dfrac{900\mathrm{e}^{\frac{1}{4}t}}{7+3\mathrm{e}^{\frac{1}{4}t}}\ \Rightarrow\ N=\dfrac{900}{3+7\mathrm{e}^{-0.25t}}$ *	A1*	1.1b
		(4)	
(b) Way 4	$N(3+7\mathrm{e}^{-0.25t})=900\ \Rightarrow \mathrm{e}^{-0.25t}=\dfrac{1}{7}\left(\dfrac{900}{N}-3\right)\ \Rightarrow\ \mathrm{e}^{-0.25t}=\dfrac{900-3N}{7N}$	M1	2.1
	$\Rightarrow t=-4\big(\ln(900-3N)-\ln(7N)\big)$ $\Rightarrow \dfrac{\mathrm{d}t}{\mathrm{d}N}=-4\left(\dfrac{-3}{900-3N}-\dfrac{7}{7N}\right)$	A1	1.1b
	$\dfrac{\mathrm{d}t}{\mathrm{d}N}=4\left(\dfrac{1}{300-N}+\dfrac{1}{N}\right)\ \Rightarrow\ \dfrac{\mathrm{d}t}{\mathrm{d}N}=4\left(\dfrac{N+300-N}{N(300-N)}\right)$	dM1	2.1
	$\dfrac{\mathrm{d}t}{\mathrm{d}N}=\left(\dfrac{1200}{N(300-N)}\right)\ \Rightarrow\ \dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	

Notes for Question 14 Continued	
(b) Way 3	
M1:	Separates the variables, an attempt to form and apply partial fractions and integrates to give $\ln \text{ terms} = kt \ \{+c\}, \ \ k \neq 0$, with or without a constant of integration c
A1:	$\frac{1}{300}\ln N - \frac{1}{300}\ln(300 - N) = \frac{1}{1200}t \ \ \{+c\}$ or equivalent with or without a constant of integration c
dM1:	Uses $t = 0, \ N = 90$ to find their constant of integration and obtains an expression of the form $\lambda e^{\frac{1}{4}t} = f(N); \ \lambda \neq 0$ or $\lambda e^{-\frac{1}{4}t} = f(N); \ \lambda \neq 0$
A1*:	Correct manipulation leading to $N = \frac{900}{3 + 7e^{-0.25t}}$ *
(b) Way 4	
M1:	Valid attempt to make t the subject, followed by an attempt to find two ln derivatives, condoning sign errors and constant errors.
A1:	$\frac{dt}{dN} = -4\left(\frac{-3}{900 - 3N} - \frac{7}{7N}\right)$ or equivalent
dM1:	Forms a common denominator to combine their fractions
A1*:	Correct algebra leading to $\frac{dN}{dt} = \frac{N(300 - N)}{1200}$ *

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